Lab 11 Quiz Review Solutions

① let μ = average daily sugar intake of at-risk population, $n = 37$, $\bar{x} = 80$, $sd = 11$

Hypotheses: $H_0: \mu = 77$ vs $H_a: \mu \neq 77$

Test Statistic: $t = \frac{\bar{x} - \mu}{SE}$ where $SE = \frac{sd}{\sqrt{n}} = \frac{11}{\sqrt{37}} = 1.8084$

$= \frac{80-77}{1.8084} = 1.66$ with $df = 36$

Approximate p-value: $df = 36 \approx 35$ so we will use $df = 35$ on the table

$1.47 < 1.66 < 1.69 \Rightarrow 0.1 < \text{p-value} < 0.15$

Conclusion: Regardless of where the exact p-value is in the above range, there is no evidence that the average daily sugar intake of the at-risk population differs from the general general population's average.

② $\mu = 90$ $sd = 8$

a) $z = \frac{95-90}{8} = 0.625$, we want $P(Z > 0.625)$

On the table $P(Z > z) = 1 - P(Z < z)$

So, $P(Z > 0.625) = 1 - P(Z < 0.625)$

$= 1 - 0.736$

$= 0.264$, 26.4%

b) we want $P(X=3)$ where X = the # of people in the sample of 10 with cholesterol > 95

$\Rightarrow$ use Binomial dist with $n=10$, $\pi = 0.264$

$P(X=3) = \binom{10}{3}(0.264)^3(0.736)^7$

$= 0.2583$, 25.83%

c) $se = \frac{8}{\sqrt{10}} = 2.5298$

$z = \frac{95-90}{2.5298} = 1.98$, we want $P(z > 1.98)$

$P(z > 1.98) = 1 - P(z < 1.98)$

$= 1 - 0.976$

$= 0.024$, 2.4%

d) We want $P(X \geq 1)$ where X = the # of sample means > 95

⇒ use Binomial dist with $n = 5$, $\pi = 0.024$

$P(X \geq 1) = 1 - P(X = 0)$

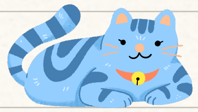
$= 1 - \binom{5}{0}(0.024)^0(0.976)^5$

$= 1 - 0.8856$

$= 0.1144$, 11.44%

③ let π = proportion of cats that prefer inside, $\hat{\pi} = \frac{155}{235} = 0.6596$, $n = 235$

Hypotheses: $\pi_0 = 0.5$ vs $H_a: \pi_0 \neq 0.5$



Test Statistic: $z = \frac{\hat{\pi} - \pi_0}{SE}$ where $SE = \sqrt{\frac{0.5(1-0.5)}{235}} = 0.0326$

$= \frac{0.6596 - 0.5}{0.0326}$

$= 4.9$

Approximate p-value: $P(z > 4.9) = 1 - P(z < 4.9) =$ nearly 0

95 CI : $\hat{\pi} \pm 1.96 SE = 0.6596 \pm 1.96\sqrt{\frac{(0.6595)(1-0.6595)}{235}}$

$= (0.599, 0.72)$

Conclusion: There is strong evidence that the proportion of cats that prefer inside differs from the proportion of cats that prefer outside. Since the CI is entirely above 0.5, we can conclude that there is evidence that the proportion of cats that prefer inside is greater than the proportion of cats that prefer outside.

④ let μ_{diff} = the average difference in resting heart rate based on after-before

$\bar{X} = -4.2$, $SE = \frac{3.4}{\sqrt{9}} = 1.133$

a) Hypotheses: $H_0: \mu_{diff} = 0$ vs $H_a: \mu_{diff} \neq 0$

Test Statistic: $t = \frac{-4.2-0}{1.133} = -3.77$ with $df = 8$

Approximate p-value: $3.36 < |-4.15| < 3.83 \Rightarrow 0.005 <$ p-value < 0.01

Conclusion: Regardless of where the exact p-value is in the above range, there is evidence of a difference between the average difference between the foxes' resting heart rate before and after the supplement. Since $\bar{X}_{diff}$ is less than 0 and the difference was constructed using after-before, there is evidence that the foxes' resting heart rate was lower after taking the supplement.

b) 95% CI: $\bar{X}_{diff} \pm 2.31 SE = -4.2 \pm 2.31(1.133)$

$= (-6.82, -1.58)$

Note that this interval matches the conclusion in part a.

⑤ a) The variability of the sample mean will be less than the variability of the population mean.

sample mean variability $= SE = \frac{9.25}{\sqrt{20}} = 2.068$, $2.068 <$ pop. variability of 9.24

b) We want $P(z_1 < Z < z_2)$

$z_1 = \frac{29-29.5}{2.068} = -0.24$ $\quad$ $z_2 = \frac{31-29.5}{2.068} = 0.73$

$P(-0.242 < Z < 0.725) = P(Z < 0.73) - P(Z < -0.24)$

$= 0.767 - 0.405$

$= 0.362$, 36.2%

c) middle 50% of data $= \bar{X} \pm Z_{0.25}\,SE$ so we need $\pm Z_{0.25}$ (or $\pm Z_{0.75}$)

$\pm Z_{0.25} \approx \pm 0.675$

So, middle 50% of data $= 29.5 \pm 0.675(2.068)$

$= (28.1, 30.9)$

d) Use the t distribution!

95% CI $= \bar{X} \pm t_{0.05,df}\,SE$ where df = 19 so $t_{0.05,19} \approx 2.09$

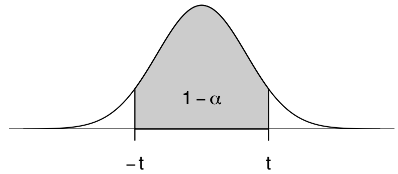
$= 30.1 \pm 2.09\left(\frac{8.95}{\sqrt{20}}\right)$

$= (25.92, 34.28)$

Interpretation: We are 95% confident that the true average level of albumin in cerebrospinal fluid is between 25.92 mg/dl and 34.28 mg/dl.

e) The width of the CI in part d depends on sample size, the critical value, and the sd. t-distributions have larger critical values and fatter tails leading to wider intervals.

Values ($\pm t$) that contain the middle $100(1-\alpha)\%$ of Student's curve with the specified degrees of freedom



						α							
df	0.25	0.2	0.15	0.1	0.05	0.04	0.03	0.02	0.01	0.005	0.001	0.0005	0.0001
1	2.41	3.08	4.17	6.31	12.7	15.9	21.2	31.8	63.7	127	637	1273	6366
2	1.60	1.89	2.28	2.92	4.30	4.85	5.64	6.96	9.92	14.1	31.6	44.7	100
3	1.42	1.64	1.92	2.35	3.18	3.48	3.90	4.54	5.84	7.45	12.9	16.3	28.0
4	1.34	1.53	1.78	2.13	2.78	3.00	3.30	3.75	4.60	5.60	8.61	10.3	15.5
5	1.30	1.48	1.70	2.02	2.57	2.76	3.00	3.36	4.03	4.77	6.87	7.98	11.2
6	1.27	1.44	1.65	1.94	2.45	2.61	2.83	3.14	3.71	4.32	5.96	6.79	9.08
7	1.25	1.41	1.62	1.89	2.36	2.52	2.71	3.00	3.50	4.03	5.41	6.08	7.88
8	1.24	1.40	1.59	1.86	2.31	2.45	2.63	2.90	3.36	3.83	5.04	5.62	7.12
9	1.23	1.38	1.57	1.83	2.26	2.40	2.57	2.82	3.25	3.69	4.78	5.29	6.59
10	1.22	1.37	1.56	1.81	2.23	2.36	2.53	2.76	3.17	3.58	4.59	5.05	6.21
11	1.21	1.36	1.55	1.80	2.20	2.33	2.49	2.72	3.11	3.50	4.44	4.86	5.92
12	1.21	1.36	1.54	1.78	2.18	2.30	2.46	2.68	3.05	3.43	4.32	4.72	5.69
13	1.20	1.35	1.53	1.77	2.16	2.28	2.44	2.65	3.01	3.37	4.22	4.60	5.51
14	1.20	1.35	1.52	1.76	2.14	2.26	2.41	2.62	2.98	3.33	4.14	4.50	5.36
15	1.20	1.34	1.52	1.75	2.13	2.25	2.40	2.60	2.95	3.29	4.07	4.42	5.24
20	1.18	1.33	1.50	1.72	2.09	2.20	2.34	2.53	2.85	3.15	3.85	4.15	4.84
25	1.18	1.32	1.49	1.71	2.06	2.17	2.30	2.49	2.79	3.08	3.73	4.00	4.62
30	1.17	1.31	1.48	1.70	2.04	2.15	2.28	2.46	2.75	3.03	3.65	3.90	4.48
35	1.17	1.31	1.47	1.69	2.03	2.13	2.26	2.44	2.72	3.00	3.59	3.84	4.39
40	1.17	1.30	1.47	1.68	2.02	2.12	2.25	2.42	2.70	2.97	3.55	3.79	4.32
45	1.17	1.30	1.46	1.68	2.01	2.12	2.24	2.41	2.69	2.95	3.52	3.75	4.27
50	1.16	1.30	1.46	1.68	2.01	2.11	2.23	2.40	2.68	2.94	3.50	3.72	4.23
55	1.16	1.30	1.46	1.67	2.00	2.10	2.23	2.40	2.67	2.92	3.48	3.70	4.20
60	1.16	1.30	1.46	1.67	2.00	2.10	2.22	2.39	2.66	2.91	3.46	3.68	4.17
65	1.16	1.29	1.46	1.67	2.00	2.10	2.22	2.39	2.65	2.91	3.45	3.66	4.15
70	1.16	1.29	1.46	1.67	1.99	2.09	2.22	2.38	2.65	2.90	3.44	3.65	4.13
75	1.16	1.29	1.45	1.67	1.99	2.09	2.21	2.38	2.64	2.89	3.43	3.64	4.11
80	1.16	1.29	1.45	1.66	1.99	2.09	2.21	2.37	2.64	2.89	3.42	3.63	4.10
85	1.16	1.29	1.45	1.66	1.99	2.09	2.21	2.37	2.63	2.88	3.41	3.62	4.08
90	1.16	1.29	1.45	1.66	1.99	2.08	2.21	2.37	2.63	2.88	3.40	3.61	4.07
95	1.16	1.29	1.45	1.66	1.99	2.08	2.20	2.37	2.63	2.87	3.40	3.60	4.06
100	1.16	1.29	1.45	1.66	1.98	2.08	2.20	2.36	2.63	2.87	3.39	3.60	4.05
200	1.15	1.29	1.45	1.65	1.97	2.07	2.19	2.35	2.60	2.84	3.34	3.54	3.97
400	1.15	1.28	1.44	1.65	1.97	2.06	2.18	2.34	2.59	2.82	3.32	3.51	3.93
600	1.15	1.28	1.44	1.65	1.96	2.06	2.18	2.33	2.58	2.82	3.31	3.50	3.92
800	1.15	1.28	1.44	1.65	1.96	2.06	2.17	2.33	2.58	2.81	3.30	3.50	3.91
1000	1.15	1.28	1.44	1.65	1.96	2.06	2.17	2.33	2.58	2.81	3.30	3.49	3.91
∞	1.15	1.28	1.44	1.64	1.96	2.05	2.17	2.33	2.58	2.81	3.29	3.48	3.89

Area to the left of z under the normal curve. The rows represent the tens digit of z and the columns represent the hundreds digit. So, for instance, to find the area to the left of 1.23, go down to the "1.2" row and over to the ".03" column, and find the answer: 0.891.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-3.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
-2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
-2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
-2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
-2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
-2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
-2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
-2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
-2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
-2	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
-1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
-1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
-1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
-1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
-1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
-1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
-1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
-1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
-1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
-1	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
-0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
-0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
-0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
-0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
-0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
-0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
-0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
-0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
-0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
-0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.5	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.6	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.7	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.8	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
2.9	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

4.9 → 1.000

Some convenient quantiles:

α	z
0.01	2.58
0.05	1.96
0.1	1.64
0.2	1.28
0.25	1.15