

## Lab 9 Practice Problems

①

Let  $M$  = the average IQ of children

Hypotheses:  $H_0: M = 95$  vs  $H_a: M \neq 95$  note that this is a two-sided test

From the data set we have  $\hat{M} = 91.0806$   $SD = 14.4039$   $n = 124$

Test Statistic:  $t = \frac{91.0806 - 95}{\frac{14.4039}{\sqrt{124}}} \approx -3.03$  with  $df = 124 - 1 = 123$

Approximate p-value:  $df = 123 \approx 100$  and we have  $|t| = 3.03$

Looking at the row corresponding to  $df = 100$ , our  $t$  falls between 2.87 and

3.39. These  $t$  values correspond to  $\alpha$  values of 0.005 and 0.001.

So, 2.87  $<$   $3.03$   $<$  3.39  $\Rightarrow$  0.001  $<$  p-value  $<$  0.005.

Regardless of where the p-value is in (0.001, 0.005) it is pretty small, and

so we reject the  $H_0$ .

95% CI:  $\alpha = 0.05$ , and  $df \approx 100$  so  $t_{0.05, 100} = \pm 1.98$

CI =  $91.0806 \pm 1.98 \left( \frac{14.4039}{\sqrt{124}} \right) = (88.54, 93.62)$

Conclusion: This study provided evidence that children's average IQ

differs from 95. Since the confidence interval is entirely below 95

there is evidence the true average for children's IQ is less than

the hypothesized value of 95.

② Let  $\pi$  = proportion of patients that tested positive on the new test

$$a) \hat{\pi} = \frac{147}{200} = 0.735 \quad SD = \sqrt{0.735(1-0.735)} = 0.4413 \quad SE = \frac{0.4413}{\sqrt{200}} = 0.0312$$

$$\alpha = 0.05 \quad \text{so} \quad Z = \pm 1.96$$

$$95\% \text{ CI} = 0.735 \pm 1.96(0.0312) = (0.6738, 0.7962)$$

b) Hypotheses:  $H_0: \pi = 0.68$  vs  $H_a: \pi \neq 0.68$

$$\text{Test Statistic: } \pi_0 = 0.68 \quad SD_0 = \sqrt{0.68(1-0.68)} = 0.4665 \quad SE_0 = \frac{0.4665}{\sqrt{200}} = 0.033$$

$$Z = \frac{0.735 - 0.68}{0.033} = 1.67$$

$$\text{Approximate p-value: } 2P(Z > 1.67) = 2P(Z < -1.67)$$

$$= 2(0.048)$$

$$= 0.096$$

Conclusion: Since this p-value is pretty big this study does not provide evidence that the new test's sensitivity differs from the old.

c)  $H_0: \pi = 0.68$  Test done in lab on R.

③  $\mu = 52 \quad \sigma = 5$

$$a) Z_1 = \frac{47 - 52}{5} = -1 \quad Z_2 = \frac{56 - 52}{5} = 0.8$$

$$P(-1 < Z < 0.8) = P(Z < 0.8) - P(Z < -1)$$

$$= 0.788 - 0.159$$

$$= 0.629$$

Approximately 62.9% of individuals scored between a 47 and a 56.

$$b) \text{ When } n=9 \quad SE = \frac{5}{\sqrt{9}} = 1.67$$

$$z = \frac{60-52}{1.67} = 4.8$$

$$P(Z > 4.8) = 1 - P(Z < 4.8) \approx 0$$

If a sample had 9 participants the probability the average test score is greater than 60 is nearly 0%.

$$c) z = \frac{43-52}{5} = -1.8$$

$$P(Z < -1.8) = 0.036$$

3.6% of patients can be considered for the alternative treatment.

$$d) P(Z > z) = 1 - P(Z < z) = 1 - 0.271 = 0.729 \quad \text{when } z = 0.61$$

$$z = \frac{\hat{\mu} - \mu}{\sigma} \Rightarrow \hat{\mu} = \mu + z\sigma$$

$$= 52 + (0.61)5$$

$$= 55.05$$

27.1% of patients scored above a 55.05.

e) Use the binomial distribution with  $n=25$  and  $\pi=0.036$

$$P(X \geq 2) = 1 - P(X=1) - P(X=0)$$

$$= 1 - \left[ \binom{25}{1} (0.036)^1 (1-0.036)^{24} \right] - \left[ \binom{25}{0} (0.036)^0 (1-0.036)^{25} \right]$$

$$\approx 0.225$$

In a sample size of 25 there is a 22.5% chance at least 2 patients will be considered for the alternative treatment.

$$\textcircled{4} \mu = 18 \quad \sigma = 4 \quad n = 4$$

a) Done in lab

$$\text{b) } SE = \frac{4}{\sqrt{4}} = 2 \quad \text{so } z = \frac{16-18}{2} = -1$$

$$P(Z < -1) = 0.159$$

The probability that this year's average sample weight is less than 16 lbs is approximately 15.9%.

$$\text{c) } z = \frac{21-18}{2} = 1.5$$

$$P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.933 = 0.067$$

The probability that this year's average sample weight is greater than 21 lbs is approximately 6.7%.

d) This is a binomial problem with  $n=5$ , but we don't currently know  $\pi$ .

$$\pi = P(z_1 < Z < z_2) \quad \text{where } z_1 = \frac{14-18}{2} = -2 \quad \text{and } z_2 = \frac{20-18}{2} = 1$$

$$\text{So, } \pi = P(-2 < Z < 1)$$

$$= P(Z < 1) - P(Z < -2)$$

$$= 0.841 - 0.023$$

$$= 0.818$$

We can now use the binomial distribution.

$$P(X \geq 2) = 1 - P(X=1) - P(X=0)$$

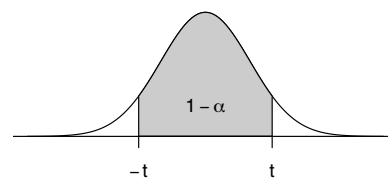
$$= 1 - \left[ \binom{5}{1} (0.818)^1 (1-0.818)^4 \right] - \left[ \binom{5}{0} (0.818)^0 (1-0.818)^5 \right]$$

$$\approx 0.9954$$

99.54%

# Student's t Table

Values ( $\pm t$ ) that contain the middle  $100(1 - \alpha)\%$  of Student's curve with the specified degrees of freedom



df	$\alpha$												
	0.25	0.2	0.15	0.1	0.05	0.04	0.03	0.02	0.01	0.005	0.001	0.0005	0.0001
1	2.41	3.08	4.17	6.31	12.7	15.9	21.2	31.8	63.7	127	637	1273	6366
2	1.60	1.89	2.28	2.92	4.30	4.85	5.64	6.96	9.92	14.1	31.6	44.7	100
3	1.42	1.64	1.92	2.35	3.18	3.48	3.90	4.54	5.84	7.45	12.9	16.3	28.0
4	1.34	1.53	1.78	2.13	2.78	3.00	3.30	3.75	4.60	5.60	8.61	10.3	15.5
5	1.30	1.48	1.70	2.02	2.57	2.76	3.00	3.36	4.03	4.77	6.87	7.98	11.2
6	1.27	1.44	1.65	1.94	2.45	2.61	2.83	3.14	3.71	4.32	5.96	6.79	9.08
7	1.25	1.41	1.62	1.89	2.36	2.52	2.71	3.00	3.50	4.03	5.41	6.08	7.88
8	1.24	1.40	1.59	1.86	2.31	2.45	2.63	2.90	3.36	3.83	5.04	5.62	7.12
9	1.23	1.38	1.57	1.83	2.26	2.40	2.57	2.82	3.25	3.69	4.78	5.29	6.59
10	1.22	1.37	1.56	1.81	2.23	2.36	2.53	2.76	3.17	3.58	4.59	5.05	6.21
11	1.21	1.36	1.55	1.80	2.20	2.33	2.49	2.72	3.11	3.50	4.44	4.86	5.92
12	1.21	1.36	1.54	1.78	2.18	2.30	2.46	2.68	3.05	3.43	4.32	4.72	5.69
13	1.20	1.35	1.53	1.77	2.16	2.28	2.44	2.65	3.01	3.37	4.22	4.60	5.51
14	1.20	1.35	1.52	1.76	2.14	2.26	2.41	2.62	2.98	3.33	4.14	4.50	5.36
15	1.20	1.34	1.52	1.75	2.13	2.25	2.40	2.60	2.95	3.29	4.07	4.42	5.24
20	1.18	1.33	1.50	1.72	2.09	2.20	2.34	2.53	2.85	3.15	3.85	4.15	4.84
25	1.18	1.32	1.49	1.71	2.06	2.17	2.30	2.49	2.79	3.08	3.73	4.00	4.62
30	1.17	1.31	1.48	1.70	2.04	2.15	2.28	2.46	2.75	3.03	3.65	3.90	4.48
35	1.17	1.31	1.47	1.69	2.03	2.13	2.26	2.44	2.72	3.00	3.59	3.84	4.39
40	1.17	1.30	1.47	1.68	2.02	2.12	2.25	2.42	2.70	2.97	3.55	3.79	4.32
45	1.17	1.30	1.46	1.68	2.01	2.12	2.24	2.41	2.69	2.95	3.52	3.75	4.27
50	1.16	1.30	1.46	1.68	2.01	2.11	2.23	2.40	2.68	2.94	3.50	3.72	4.23
55	1.16	1.30	1.46	1.67	2.00	2.10	2.23	2.40	2.67	2.92	3.48	3.70	4.20
60	1.16	1.30	1.46	1.67	2.00	2.10	2.22	2.39	2.66	2.91	3.46	3.68	4.17
65	1.16	1.29	1.46	1.67	2.00	2.10	2.22	2.39	2.65	2.91	3.45	3.66	4.15
70	1.16	1.29	1.46	1.67	1.99	2.09	2.22	2.38	2.65	2.90	3.44	3.65	4.13
75	1.16	1.29	1.45	1.67	1.99	2.09	2.21	2.38	2.64	2.89	3.43	3.64	4.11
80	1.16	1.29	1.45	1.66	1.99	2.09	2.21	2.37	2.64	2.89	3.42	3.63	4.10
85	1.16	1.29	1.45	1.66	1.99	2.09	2.21	2.37	2.63	2.88	3.41	3.62	4.08
90	1.16	1.29	1.45	1.66	1.99	2.08	2.21	2.37	2.63	2.88	3.40	3.61	4.07
95	1.16	1.29	1.45	1.66	1.99	2.08	2.20	2.37	2.63	2.87	3.40	3.60	4.06
100	1.16	1.29	1.45	1.66	1.98	2.08	2.20	2.36	2.63	2.87	3.39	3.60	4.05
200	1.15	1.29	1.45	1.65	1.97	2.07	2.19	2.35	2.60	2.84	3.34	3.54	3.97
400	1.15	1.28	1.44	1.65	1.97	2.06	2.18	2.34	2.59	2.82	3.32	3.51	3.93
600	1.15	1.28	1.44	1.65	1.96	2.06	2.18	2.33	2.58	2.82	3.31	3.50	3.92
800	1.15	1.28	1.44	1.65	1.96	2.06	2.17	2.33	2.58	2.81	3.30	3.50	3.91
1000	1.15	1.28	1.44	1.65	1.96	2.06	2.17	2.33	2.58	2.81	3.30	3.49	3.91
$\infty$	1.15	1.28	1.44	1.64	1.96	2.05	2.17	2.33	2.58	2.81	3.29	3.48	3.89

# Normal Table - table gives $P(Z < z)$

Area to the left of  $z$  under the normal curve. The rows represent the tens digit of  $z$  and the columns represent the hundreds digit. So, for instance, to find the area to the left of 1.23, go down to the "1.2" row and over to the ".03" column, and find the answer: 0.891.

$z$	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-3.3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
-3.2	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-3.1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-3	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
-2.9	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
-2.8	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
-2.7	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
-2.6	0.005	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
-2.5	0.006	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005	0.005
-2.4	0.008	0.008	0.008	0.008	0.007	0.007	0.007	0.007	0.007	0.006
-2.3	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.008
-2.2	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.012	0.011	0.011
-2.1	0.018	0.017	0.017	0.017	0.016	0.016	0.015	0.015	0.015	0.014
-2	0.023	0.022	0.022	0.021	0.021	0.020	0.020	0.019	0.019	0.018
-1.9	0.029	0.028	0.027	0.027	0.026	0.026	0.025	0.024	0.024	0.023
-1.8	0.036	0.035	0.034	0.034	0.033	0.032	0.031	0.031	0.030	0.029
-1.7	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
-1.6	0.055	0.054	0.053	0.052	0.051	0.049	0.048	0.047	0.046	0.046
-1.5	0.067	0.066	0.064	0.063	0.062	0.061	0.059	0.058	0.057	0.056
-1.4	0.081	0.079	0.078	0.076	0.075	0.074	0.072	0.071	0.069	0.068
-1.3	0.097	0.095	0.093	0.092	0.090	0.089	0.087	0.085	0.084	0.082
-1.2	0.115	0.113	0.111	0.109	0.107	0.106	0.104	0.102	0.100	0.099
-1.1	0.136	0.133	0.131	0.129	0.127	0.125	0.123	0.121	0.119	0.117
-1	0.159	0.156	0.154	0.152	0.149	0.147	0.145	0.142	0.140	0.138
-0.9	0.184	0.181	0.179	0.176	0.174	0.171	0.169	0.166	0.164	0.161
-0.8	0.212	0.209	0.206	0.203	0.200	0.198	0.195	0.192	0.189	0.187
-0.7	0.242	0.239	0.236	0.233	0.230	0.227	0.224	0.221	0.218	0.215
-0.6	0.274	0.271	0.268	0.264	0.261	0.258	0.255	0.251	0.248	0.245
-0.5	0.309	0.305	0.302	0.298	0.295	0.291	0.288	0.284	0.281	0.278
-0.4	0.345	0.341	0.337	0.334	0.330	0.326	0.323	0.319	0.316	0.312
-0.3	0.382	0.378	0.374	0.371	0.367	0.363	0.359	0.356	0.352	0.348
-0.2	0.421	0.417	0.413	0.409	0.405	0.401	0.397	0.394	0.390	0.386
-0.1	0.460	0.456	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425
0	0.500	0.496	0.492	0.488	0.484	0.480	0.476	0.472	0.468	0.464
0.1	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.2	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.3	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.4	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.5	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.6	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.7	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.8	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.9	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
1	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1.1	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.2	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.3	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.4	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.5	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.6	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.7	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.8	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.9	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
2	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2.1	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.2	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.3	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.4	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.5	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994
2.6	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.995	0.995	0.995
2.7	0.995	0.995	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.996
2.8	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
2.9	0.997	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999	0.999
3.1	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3.3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Some convenient quantiles:

$\alpha$	$z$
0.01	2.58
0.05	1.96
0.1	1.64
0.2	1.28
0.25	1.15

