BST 760: Advanced Regression
Breheny

Assignment 3
Due: Thursday, February 7

1. Consider the gamma distribution, which has support on $[0, \infty)$:

$$f(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y},$$

with mean α/β and variance α/β^2.

   (a) Reparameterize the gamma distribution in terms of μ and α, where $\mu = \mathrm{E}(Y)$. Use this reparameterization for (b)-(j) below.

   (b) Show that the gamma distribution belongs to the exponential family; in particular,
      (i) What is θ, as a function of μ?
      (ii) What is $b(\theta)$?
      (iii) What is ϕ?
      (iv) What is $c(y, \phi)$?

   (c) Find $b'(\theta)$.

   (d) Write down the score for an observation Y_i.

   (e) What is the MLE of θ?

   (f) Find $b''(\theta)$.

   (g) Find $W(\mu)$ and show that $\mathrm{Var}(Y)$ can be written as $\phi W(\mu)$.

   (h) What is the information for an observation from the gamma distribution?

   (i) What is the information for a sample of n independent observations from the gamma distribution?

   (j) What is the canonical link for the gamma distribution?